

Simplified construction and physical realization of n -qubit controlled phase gates

Shi-Biao Zheng

Department of Physics

Fuzhou University

Fuzhou 350002, P. R. China

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We show that with the assistance of a third level of the qubits an n -qubit phase gate can be constructed from $2n - 4$ two-qutrit conditional swap gates, a single qutrit-qubit controlled phase gate, and two single-qutrit operations. Unlike previous schemes, our scheme uses the auxiliary level to "expose" some state to the qutrit-qubit controlled phase gate, instead of using it to "hide" states from the conditional dynamics. Neither the number of the additional levels nor that of single-qutrit operations needs to increase with n . We propose a physical implementation of the required elementary gates in cavity QED, and show that the total gate time may be greatly reduced as compared with that required in the previous methods.

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I. INTRODUCTION

A quantum computer, taking advantage of superposition and entanglement, could realize additional information processing functions. Since Shor discovered that a quantum computer could efficiently factorize large integers in 1994 [1], quantum computation has become a truly interdisciplinary field across physics, information science, and engineering. In a quantum computer information is stored in quantum bits (qubits) which are represented by two-level systems, such as atoms and ions. The building blocks of a quantum computer are logic gates and any quantum computational network can be decomposed into a series of two-qubit plus one-qubit logic gates [2,3].

For the implementation of a practical quantum computational task, a large number of qubits should be involved and controlled quantum gates among these qubits are required. Of particular importance is the n -qubit controlled phase gate that shifts the phase of one and only one of the state components. This gate is an essential ingredient for implementation of quantum algorithms [1,4] and quantum Fourier transform [5]. By placing the Hadamard gates before and after the three-qubit controlled π -phase gate on one of the qubits one can implement the Toffoli gate that inverts the state of the target qubit conditional on the state of the two control qubits. In addition, such phase gates are useful for the implementation of quantum error correction [6-12]. The Toffoli gate has been demonstrated in nuclear magnetic resonance [7], linear optics [13], ion trap [14], and circuit QED systems [12,15], however the controlled phase gates involving more than three qubits has not been experimentally implemented. Though an n -qubit controlled phase gate could be decomposed into the elementary one- and two-qubit gates, it would be extremely complex and difficult to solve a practical problem, for example, a search with a quantum computer for an item from a disordered system [4], via such a decomposition. On one hand, the number of needed logic operations exponentially increases with the number of qubits. On the other hand, a quantum system is very fragile and may be destroyed by decoherence arising from the coupling with the environment. The error of performance increases as the number of logic gates increases.

Recently, a highly efficient scheme has been proposed for implementation of Grover's search algorithm in the trapped-ion system using Householder reflections [16]. The distinct feature of this scheme is that each of the inversion-about-average operation and the oracle query can be realized in a single step so that the physical implementation of each logic iteration is significantly simplified as compared with the methods based on decomposition of multi-qubit controlled phase gates. However, the scheme requires the ions to be initially prepared in the entangled Dicke state, which is experimentally demanding. The search algorithm can also be simplified using qudits [17]. Despite these advances, many theoretical and experimental endeavors are still being directed toward the realization of multi-qubit controlled phase gates for their applications in Shor's algorithm and quantum error correction. So far implementation of the Toffoli gate and three-qubit controlled phase gate based on one- and two-qubit gates has not been reported due to decoherence, and it is of importance to simplify the realization of a multi-qubit gate so that the number of required elementary operations does not exponentially increase with the number of qubits. Resch et al. have shown that the number of two-qubit gates required to implement a Toffoli gate acting on three qubits can be reduced if one of the three qubits has a third state that is accessible during the gate operation [18]. The basic idea of the method is to "hide" certain states from the two-qubit controlled phase gate. The technique can be generalized to higher-order Toffoli gate with n qubits by making the target qubit an n -level qudit. In general, this method requires one two-qubit controlled phase gate, $2n - 4$ controlled-NOT gates, and $2(n - 2)$ single-qudit gates to construct an n -qubit controlled phase gate. The limitation of the method is that it requires the number of the accessible states of the target to equal the number of the qubits involved in the gate operation, which is experimentally problematic since it may be difficult

to get as many states with long coherence times as required in an realistic physical system.

In this paper we show that, with the assistance of an auxiliary state, an n -qubit quantum phase gate could be constructed from $2n - 4$ two-qutrit conditional swap gates, a single qutrit-qubit controlled phase gate, and two single-qutrit gates. Unlike the previous methods [12-15,18,19], the auxiliary state is used to "expose" some state to the two-qutrit controlled phase gate, not to "hide" certain states. In comparison with the method of Ref. [18], the present one does not require the number of the available states of the target increase with n . For each qubit only one additional state is required to be accessible during the gate operation. Furthermore, for certain realistic physical systems it is easier to implement the conditional swap gate than the controlled-NOT gate. Another advantage of the present method is that the number of required single-qutrit operations is independent of n . We propose an experimental realization of the two-qutrit conditional swap gate and qutrit-qubit controlled phase gate in cavity QED. During the operation the atomic qubits are always in the ground states and the cavity mode is only virtually excited and thus the scheme is insensitive to both the atomic spontaneous emission and cavity decay. The scheme is generic and can be implemented in other physical systems in which the qubits have an auxiliary state.

The paper is organized as follows. In Sec.2, we describe the method to construct the n -qubit controlled phase gate using only qutrits, and show that the number of required single-qutrit gates does not increase with n . In Sec.3, as an example for the physical implementation of this method we demonstrate that the required elementary gates can be realized in the context of cavity QED. The cavity mode, together with external classical fields, can induce the controlled atom-atom coupling. It is shown that in this system the implementation of the conditional swap gate is easier than that of the controlled-NOT gate and the gate time is reduced. The conclusion appears in Sec.4.

II. CONSTRUCTION OF THE N-QUBIT CONTROLLED PHASE GATE WITH QUTRITS

We first consider a three-qubit system. The computational basis states of each qubit is represented by $|1\rangle$ and $|0\rangle$. Meanwhile, each qubit has an auxiliary state $|a\rangle$. The main ingredients for constructing the three-qubit controlled phase gate are the two-qutrit conditional swap gate $U_{j,k} = e^{\pi(|1_j a_k\rangle\langle a_j 1_k| - |a_j 1_k\rangle\langle 1_j a_k|)/2}$ and qutrit-qubit controlled phase gate $V_{j,k} = e^{i\phi|a_j 1_k\rangle\langle a_j 1_k|}$. Without loss of the generality, we assume that the three qubits are initially in the state

$$\sum_{x,y,z=0,1} \alpha_{x,y,z} |x_1 y_2 z_3\rangle. \quad (1)$$

We first perform the single-qutrit transformation L_1 on qutrit 1: $|1_1\rangle \rightarrow |a_1\rangle$, which leads to

$$\sum_{y,z=0,1} \alpha_{0,y,z} |0_1 y_2 z_3\rangle + \sum_{y,z=0,1} \alpha_{1,y,z} |a_1 y_2 z_3\rangle. \quad (2)$$

Then the gate U_{12} is performed between qutrits 1 and 2, resulting in

$$\sum_{y,z=0,1} \alpha_{0,y,z} |0_1 y_2 z_3\rangle + \sum_{z=0,1} \alpha_{1,0,z} |a_1 0_2 z_3\rangle + \sum_{z=0,1} \alpha_{1,1,z} |1_1 a_2 z_3\rangle. \quad (3)$$

Next we apply the gate V_{23} between 2 and 3 to obtain

$$\sum_{y,z=0,1} \alpha_{0,y,z} |0_1 y_2 z_3\rangle + \sum_{z=0,1} \alpha_{1,0,z} |a_1 0_2 z_3\rangle + \alpha_{1,1,0} |1_1 a_2 0_3\rangle + \alpha_{1,1,1} e^{i\phi} |1_1 a_2 1_3\rangle. \quad (4)$$

Now the gate U_{21} is again performed between 1 and 2, leading to

$$\sum_{y,z=0,1} \alpha_{0,y,z} |0_1 y_2 z_3\rangle + \sum_{z=0,1} \alpha_{1,0,z} |a_1 0_2 z_3\rangle + \alpha_{1,1,0} |a_1 1_2 0_3\rangle + \alpha_{1,1,1} e^{i\phi} |a_1 1_2 1_3\rangle. \quad (5)$$

Finally, we perform the single-qutrit transformation $M_1 : |a_1\rangle \rightarrow |1_1\rangle$ and the state becomes

$$\sum_{y,z=0,1} \alpha_{0,y,z} |0_1 y_2 z_3\rangle + \sum_{z=0,1} \alpha_{1,0,z} |1_1 0_2 z_3\rangle + \alpha_{1,1,0} |1_1 1_2 0_3\rangle + \alpha_{1,1,1} e^{i\phi} |1_1 1_2 1_3\rangle, \quad (6)$$

in which if and only if all the qubits are initially in the state $|1\rangle$ the system undergoes a phase shift ϕ . It is worthwhile mentioning that only when qubits 1 and 2 are initially in the state $|1_1 1_2\rangle$ the state of qubit 2 can be transformed

to $|a_2\rangle$ by the gate U_{12} and then qubits 2 and 3 be subjected to the controlled phase gate V_{23} which only affects the non-computational state $|a_21_3\rangle$. In other words, the auxiliary level $|a\rangle$ is used to "expose" the initial qubit state $|1_11_21_3\rangle$ to the controlled phase gate. This is distinguished from the previous schemes [12-15,18,19] in which the auxiliary levels are used to "hide" certain states so that the controlled phase gate only affects one computational state.

We note that the idea can be generalized to produce the n -qubit phase gate

$$U_n = e^{i\phi|1_11_2\dots1_n\rangle\langle1_11_2\dots1_n|} \quad (7)$$

by applying a sequence of operations: $L_1, U_{1,2}, U_{2,3}, \dots, U_{n-2,n-1}, V_{n-1,n}, U_{n-1,n-2}, U_{n-2,n-3}, \dots, U_{2,1}$, and M_1 . Therefore, $2n - 4$ two-qutrit conditional swap operations, a single qutrit-qubit controlled phase gate, and two single-qutrit operations are sufficient for the construction of an n -qubit controlled phase gate. One appealing feature of the method is that neither the number of the required additional levels nor the number of single-qutrit operations increases with n .

III. PHYSICAL IMPLEMENTATION

We consider that n identical atoms are trapped in a cavity. Each atom has one excited state $|r\rangle$ and three ground states $|1\rangle$, $|0\rangle$, and $|a\rangle$, as shown in Fig. 1. The transition $|1\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant g . For implementation of logic operations between the j th and $(j+1)$ th atoms, the transition $|a\rangle \rightarrow |r\rangle$ for each of these two atoms is driven by a classical laser field. Assume the classical field and cavity mode are detuned from the respective transitions by Δ_1 and Δ_2 , respectively. In the interaction picture, the Hamiltonian is

$$H = e^{i\Delta_1 t}(\Omega_j e^{-i\varphi_j} |r_j\rangle \langle a_j| + \Omega_{j+1} e^{-i\varphi_{j+1}} |r_{j+1}\rangle \langle a_{j+1}|) + \sum_{m=1}^n g a e^{i\Delta_2 t} |r_m\rangle \langle 1_m| + H.c., \quad (8)$$

where a is the annihilation operator of the cavity mode, and Ω_j and φ_j are the Rabi frequency and phase of the laser field driving the j th atom, respectively. Under the condition $\Delta_1, \Delta_2 \gg \Omega_j, g$ the upper level $|r\rangle$ can be adiabatically eliminated, leading to the Raman coupling of the two ground states and Stark shifts. Then the dynamics of the system is described by the effective Hamiltonian [20]

$$\begin{aligned} H_e = & -\frac{\Omega_j^2}{\Delta_1} |a_j\rangle \langle a_j| - \frac{\Omega_{j+1}^2}{\Delta_1} |a_{j+1}\rangle \langle a_{j+1}| - \lambda_j (a S_j^+ e^{i\varphi_j} e^{i\delta t} + a^\dagger S_j^- e^{-i\varphi_j} e^{-i\delta t}) \\ & - \lambda_{j+1} (a S_{j+1}^+ e^{i\varphi_{j+1}} e^{i\delta t} + a^\dagger S_{j+1}^- e^{-i\varphi_{j+1}} e^{-i\delta t}) - \sum_{m=1}^n \frac{g^2}{\Delta_2} a^\dagger a |1_m\rangle \langle 1_m|, \end{aligned} \quad (9)$$

where $\lambda_j = \frac{\Omega_j g}{2} (\frac{1}{\Delta_1} + \frac{1}{\Delta_2})$, $\delta = \Delta_2 - \Delta_1$, $S_j^+ = |a_j\rangle \langle 1_j|$, and $S_j^- = |1_j\rangle \langle a_j|$.

In the case $\delta \gg \lambda_j, \frac{\Omega_j^2}{\Delta_1}, \frac{g^2}{\Delta_2}$, there is no energy exchange between the atomic system and the cavity. The energy conserved transitions are between $|a_j 1_{j+1} n\rangle$ and $|1_j a_{j+1} n\rangle$. The effective coupling for the transition $|1_j a_{j+1} n\rangle \rightarrow |a_j 1_{j+1} n\rangle$, mediated by $|1_j 1_{j+1} n + 1\rangle$ and $|a_j a_{j+1} n - 1\rangle$ is given by [21,22]

$$\begin{aligned} & \frac{\langle a_j 1_{j+1} n | H_e | 1_j 1_{j+1} n + 1 \rangle \langle 1_j 1_{j+1} n + 1 | H_e | 1_j a_{j+1} n \rangle}{\delta} \\ & + \frac{\langle a_j 1_{j+1} n | H_e | a_j a_{j+1} n - 1 \rangle \langle a_j a_{j+1} n - 1 | H_e | 1_j a_{j+1} n \rangle}{-\delta} \\ & = \xi e^{i\varphi}, \end{aligned} \quad (10)$$

where $\xi = \frac{\lambda_j \lambda_k}{\delta}$ and $\varphi = \varphi_j - \varphi_{j+1}$. Since the two transition paths interfere destructively the effective Rabi frequency is independent of the photon-number of the cavity mode. In addition to the two-qubit coupling, the nonresonant Raman coupling leads to further Stark shift. Then we obtain the new effective Hamiltonian

$$\begin{aligned} H'_e = & (-\frac{\Omega_j^2}{\Delta_1} + \frac{\lambda_j^2}{\delta} a a^\dagger) |a_j\rangle \langle a_j| + (-\frac{\Omega_{j+1}^2}{\Delta_1} + \frac{\lambda_{j+1}^2}{\delta} a a^\dagger) |a_{j+1}\rangle \langle a_{j+1}| \\ & - \frac{\lambda_j^2}{\delta} a^\dagger a |1_j\rangle \langle 1_j| - \frac{\lambda_{j+1}^2}{\delta} a^\dagger a |1_{j+1}\rangle \langle 1_{j+1}| \\ & + \xi (e^{i\varphi} S_j^+ S_{j+1}^- + e^{-i\varphi} S_j^- S_{j+1}^+) - \sum_{m=1}^n \frac{g^2}{\Delta_2} a^\dagger a |1_m\rangle \langle 1_m|. \end{aligned} \quad (11)$$

The photon-number does not change during the process since $[a^\dagger a, H'_e] = 0$. When the cavity mode is initially in the vacuum state it will remain in the vacuum state throughout the procedure. Then the effective Hamiltonian H'_e reduces to

$$H'_e = -\mu_j |a_j\rangle \langle a_j| - \mu_{j+1} |a_{j+1}\rangle \langle a_{j+1}| + \xi(e^{i\varphi} S_j^+ S_{j+1}^- + e^{-i\varphi} S_j^- S_{j+1}^+), \quad (12)$$

where

$$\mu_j = \frac{\Omega_j^2}{\Delta_1} - \frac{\lambda_j^2}{\delta}$$

After an interaction time t we obtain the state evolution

$$\begin{aligned} |a_j 1_{j+1}\rangle &\rightarrow e^{i\mu t} \{ [\cos(\eta t) - i \frac{\epsilon}{2\eta} \sin(\eta t)] |a_j 1_{j+1}\rangle - i \frac{\xi}{\eta} e^{-i\varphi} \sin(\eta t) |1_j a_{j+1}\rangle \}, \\ |1_j a_{j+1}\rangle &\rightarrow e^{i\mu t} \{ [\cos(\eta t) + i \frac{\epsilon}{2\eta} \sin(\eta t)] |1_j a_{j+1}\rangle - i \frac{\xi}{\eta} e^{i\varphi} \sin(\eta t) |a_j 1_{j+1}\rangle \}, \\ |a_j 0_{j+1}\rangle &\rightarrow e^{i\mu_j t} |a_j 0_{j+1}\rangle. \end{aligned} \quad (13)$$

where $\mu = (\mu_j + \mu_{j+1})/2$, $\epsilon = \mu_{j+1} - \mu_j$ and $\eta = \sqrt{\xi^2 + (\mu_j - \mu_{j+1})^2/4}$. The basis states $|0_j 0_{j+1}\rangle$, $|0_j 1_{j+1}\rangle$, $|1_j 0_{j+1}\rangle$, and $|1_j 1_{j+1}\rangle$ remain unchanged during the interaction. As has been shown, before the operation $U_{j,j+1}$ ($U_{j+1,j}$) the j th and $(j+1)$ th atoms have no probability of being populated in the states $|0_j a_{j+1}\rangle$, $|1_j a_{j+1}\rangle$ ($|a_j 1_{j+1}\rangle$), and $|a_j a_{j+1}\rangle$ so that it is unnecessary to consider the evolution of these states during the gate operation $U_{j,j+1}$ ($U_{j+1,j}$). With the choice of $\mu_j = \mu_{j+1} = \mu$, $\xi t = \pi/2$ and $\varphi = -\pi/2$ ($\varphi = \pi/2$) the conditional swap operation $U_{j,j+1}$ ($U_{j+1,j}$) is obtained through the transformation (13) plus the single-qubit phase shifts: $|a_j\rangle \rightarrow e^{-i\pi\mu/2\xi} |a_j\rangle$ and $|a_{j+1}\rangle \rightarrow e^{-i\pi\mu/2\xi} |a_{j+1}\rangle$. Before the operation $V_{n-1,n}$ the $(n-1)$ th and n th atoms are not populated in the states $|0_{n-1} a_n\rangle$, $|1_{n-1} a_n\rangle$, and $|a_{n-1} a_n\rangle$. Therefore, we only need to consider the evolutions of $|a_{n-1} 1_n\rangle$ and $|a_{n-1} 0_n\rangle$. With the choice of $\eta t = \pi$, we obtain

$$\begin{aligned} |a_{n-1} 1_n\rangle &\rightarrow e^{i\pi(1+\mu/\eta)} |a_{n-1} 1_n\rangle, \\ |a_{n-1} 0_n\rangle &\rightarrow e^{i\pi\mu_{n-1}/\eta} |a_{n-1} 1_n\rangle. \end{aligned} \quad (14)$$

This transformation, together with the single-qutrit phase shifts $|a_{n-1}\rangle \rightarrow e^{-i\pi\mu_{n-1}/\eta} |a_{n-1}\rangle$, corresponds to the conditional phase operation $V_{n-1,n}$. The conditional phase shift $\pi(1 + \epsilon/2\eta)$ is controllable via the Rabi frequencies of the two classical fields.

It is worth noticing that the qutrit-qubit controlled phase gate reduces to the two-qubit controlled phase gate when the logic states of qubit $n-1$ are represented by $|0_{n-1}\rangle$ and $|a_{n-1}\rangle$ and qubit n uses $|0_n\rangle$ and $|1_n\rangle$ as the logic states [21,22]. For the present cavity QED system the duration of the two-qutrit conditional swap gate is one half of that of the two-qubit controlled π -phase gate. For the implementation of the n -qubit controlled π -phase gate in the present system the scheme of Ref. [18] requires $2n-3$ two-qubit controlled π -phase gates, $2n-4$ single-qubit gate, and $2(n-2)$ single-qutrit gates. The present method instead uses $2n-4$ two-qutrit conditional swap gates, one controlled qutrit-qubit controlled π -phase gate, and two single-qutrit gates. The total time for two-atom couplings is reduced by $(2n-4)\pi/(2\xi)$. In addition, the number of required single-atom operations is greatly reduced. The method of Ref. [15] uses a single two-qubit controlled phase gate and two qubit-qutrit swap gate to implement the three-qubit phase gate. However, the scheme can not be directly generalized to higher-order n -qubit controlled phase gates. Furthermore, the duration of the second swap gate should be three times of that of the first one.

IV. SUMMARY

In summary, we have suggested a scheme for decomposing an n -qubit controlled phase gate into $2n-4$ two-qutrit conditional swap gates, a single qutrit-qubit controlled phase gate, and two single-qutrit gates. In the scheme each qubit needs to have a single additional state that can be addressed during the gate operation. This auxiliary state is used to "expose" one and only one initial computational state to the qutrit-qubit controlled phase gate, which is distinguished from previous methods using the additional levels to "hide" one or more computational states from the two-qubit controlled phase gate. In comparison with the scheme of Ref. [18], the procedure is greatly simplified and the total gate time is reduced. We illustrate the idea in cavity QED. However, the scheme can be readily applied to other systems that have three levels with long coherence times, such as trapped ions and superconducting circuits.

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- [1] P.W.Shor, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science (IEEE Computer Society, Los Alamitos, CA, 1994), P.116.
 - [2] T. Sleator and H. Weinfurter, Phys. Rev. Lett. 74, 4087 (1995).
 - [3] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A 52, 3457 (1995).
 - [4] L. K. Grover, Phys. Rev. Lett. 79, 325 (1997); Phys. Rev. Lett. 79, 4709 (1997).
 - [5] M. A. Nielsen and I. L. Chuang, quantum computation and quantum information (Cambridge University Press, Cambridge, U. K., 2000).
 - [6] P. W. Shor, Phys. Rev. A 52, R2493 (1995).
 - [7] D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T. F. Havel, and S. S. Somaroo, Phys. Rev. Lett. 81, 2152 (1998).
 - [8] E. Knill, R. Laflamme, R. Martinez, and C. Negrevergne, Phys. Rev. Lett. 86, 5811 (2001).
 - [9] J. Chiaverini et al., Nature 432, 602 (2004).
 - [10] T. B. Pittman, B. C. Jacobs, and J. D. Franson, Phys. Rev. A 71, 052332 (2005).
 - [11] T. Aoki, G. Takahashi, T. Kajiya, Jun-ichi Yoshikawa, S. L. Braunstein, P. van Loock, and A. Furusawa, Nat. Phys. 5, 541 (2009).
 - [12] M. D. Reed, L. DiCarlo, S. E. Nigg, L. Sun, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature 482, 382 (2012).
 - [13] B. P. Lanyon et al., Nat. Phys. 5, 134 (2009).
 - [14] T. Monz, K. Kim, W. Hänsel, M. Riebe, A. S. Villar, P. Schindler, M. Chwalla, M. Hennrich, and R. Blatt, Phys. Rev. Lett. 102, 040501 (2009).
 - [15] A. Fedorov, L. Steffen, M. Baur, M. P. da Silva, and A. Wallraf, Nature 481, 170 (2012).
 - [16] S. S. Ivanov, P. A. Ivanov, I. E. Linington, and N. V. Vitanov, Phys. Rev. A 81, 042328 (2010).
 - [17] S. S. Ivanov, H. S. Tonchev, and N. V. Vitanov, Phys. Rev. A 85, 062321 (2012).
 - [18] T. C. Ralph, K. J. Resch, A. Gilchrist, Phys. Rev. A 75, 022313 (2007).
 - [19] M. Borrelli, L. Mazzola, M. Paternostro, and S. Maniscalco, Phys. Rev. A 84, 012314 (2011).
 - [20] D. F. V. James and J. Jerke, Can. J. Phys. 85, 625 (2007).
 - [21] S. B. Zheng and G. C. Guo, Phys. Rev. Lett. 85, 2392 (2000).
 - [22] L. You, X. X. Yi, and X. H. Su, Phys. Rev. A 67, 032308 (2003).

Fig. 1 (color online). The atomic level configuration and excitation scheme to realize the two-qutrit swap gate and qutrit-qubit controlled phase gate. The transition $|1\rangle \rightarrow |r\rangle$ is coupled to the cavity mode and $|a\rangle \rightarrow |r\rangle$ is driven by a classical laser field. The classical field and cavity mode are detuned from the respective transitions by Δ_1 and Δ_2 .

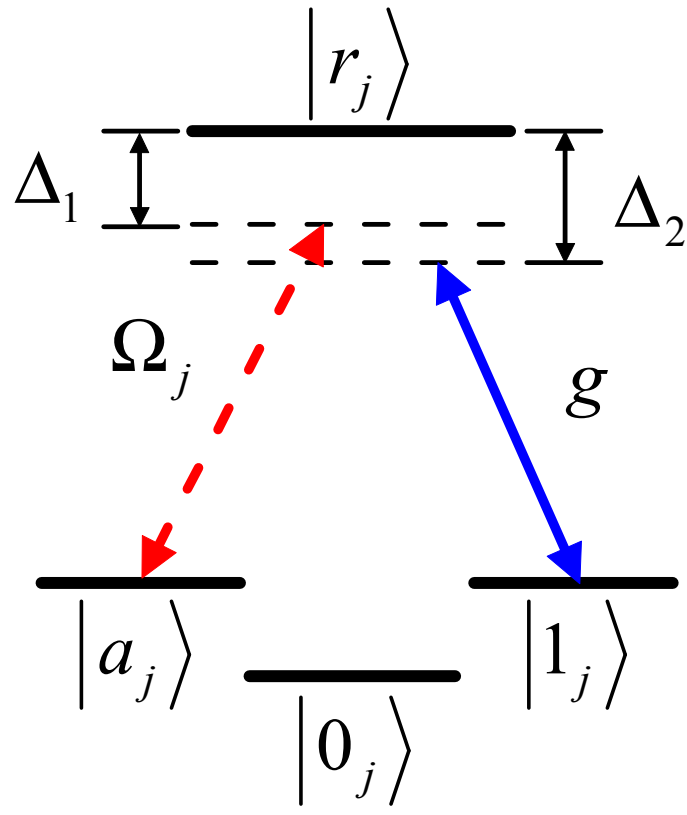


FIG. 1: